

Department of Mathematics Howard University
Applied Calculus (Math-026)- Final Examination
December 10, 2003

Do all questions. Show all work.
The time for this examination is two (2) hours.

1. [20 Points]

Use techniques of differentiation to find the derivative of each of the following:

a. $y = x^2 e^{3x}$

b. $y = \frac{x^2 - 4}{x - 2}$

c. $y = (x^2 - 6x + 2)^5$

d. $y = \ln(2x + 1)$

2. [15 Points]

If $4x^2 - 2y^2 = 9$, use implicit differentiation to find (a) $\frac{dy}{dx}$ and (b) $\frac{d^2y}{dx^2}$.
Simplify each result.

3. [10 Points]

Find the indicated limits:

a. $\lim_{x \rightarrow 1} \frac{x^2 + x - 2}{x^2 - 1}$

b. $\lim_{x \rightarrow \infty} \frac{x - 3}{2x + x^3}$

4. [10 Points]

The gross annual earnings of a certain company were $f(t) = \sqrt{10t^2 + 2t + 52}$ thousand dollars t years after its formation in January 1988. At what rate were the gross annual earnings of the company growing in January 1992?

5. [20 Points]

Let $f(x) = 2x^5 + 3x^2 - 12x + 3$.

Determine where the function is increasing, decreasing, concave upward, and concave downward. Find the relative extrema and inflection point and sketch the graph.

6. [20 Points]

The total cost, $C(x)$, of producing x units of a particular commodity is given by $C(x) = \frac{1}{4}(x^2 + 3x + 52)$ dollars and $p(x) = \frac{1}{4}(18 - x)$ dollars is the price at which all x units will be sold.

- a. Find the marginal cost and the marginal revenue.
- b. Use marginal cost to estimate the cost of producing the 3rd unit.
- c. Find the actual cost of producing the 3rd unit.
- d. Use the marginal revenue to estimate the revenue derived from the sale of the 3rd unit.

7. [20 Points]

An efficient study of the morning shift at a certain factory indicates that an average worker who arrives on the job at 8 a.m. will have produced $Q(t) = -t^3 + 6t^2 + 24t$ units t hours later.

- Compute the worker's rate of production at 11:00 a.m.
- At what rate is the worker's rate of production changing with respect to time at 11:00 a.m.?

8. [10 Points]

When electric blenders are sold for p dollars each, manufacturers will supply $S(p) = p - 10$ blenders to local retailers while the local demand will be $D(p) = \frac{5600}{p}$ blenders.

At what market price will the manufacturer's supply of electric blenders be equal to the consumer's demand for blenders? How many blenders will be sold at this price?

9. [10 points]

Use logarithmic differentiation to find $\frac{dy}{dx}$ if $y = \sqrt{\frac{2x^2+1}{x^2+2}}$. Simplify your answer.

10. [10 Points]

Use integration by substitution to find:

- $\int 2x e^{x^2+2} dx$
- $\int \frac{10x^3-5x}{\sqrt{x^4-x^2+6}} dx$

11. [10 Points]

Use integration by parts to compute:

(a) $\int x e^{2x} dx$

OR

(b) $\int x \ln 2x dx.$

12. [10 Points]

Evaluate each of the following:

a. $\int_0^3 x^2 dx$

b. $\int_1^3 \frac{1}{x} dx$

13. [10 Points]

Find the area of the region bounded by the curves $y = x^2$ and $y = 2x$. Give a sketch of the region.

14. [15 Points]

For $f(x, y) = 6x^2 - 2x^2y^2 + y^2 + 5$, find the following:

- | | | |
|--------------|--------------|--------------|
| (a) f_x | (b) f_y | (c) f_{xx} |
| (d) f_{yy} | (e) f_{xy} | (f) f_{yx} |

15. [10 Points]

Find the general solution of the differential equation $\frac{dy}{dx} = \frac{3x^2+2}{y}$.