

The Final Exam For Cal II Dec.10/2003.

Problems 2,7,9 and 10 , 20 points and the rest each 10 points.Note there are 16 problems.

1.Sketch the region enclosed by the curves $y = \sqrt{(2-x)}$, $y = -x$ and $x - axis$, then find its area.

2.Find the volume of the solid that results when the region enclosed by the curves $y = x^{\frac{3}{2}}$, $x = 4$ and x -axis rotates about (a) x -axis (b) y -axis.

3.A cylindrical tank of radius 5 ft and height 9 ft is two-thirds filled with water. Find the work required to pump the water over the upper rim.(The weight density of water is $62.4 \text{ lb}/\text{ft}^3$.)

4.Find the exact arc length of the parametric curve

$$x = \cos(t) + t \sin(t), y = \sin(t) - t \cos(t), 0 \leq t \leq \pi$$

5.Evaluate the integral $\int_0^{\frac{\pi}{2}} \sin^2 x \cos^3 x dx$.

6.Evaluate the integral $\int_0^{\frac{\pi}{2}} \tan^2(\frac{x}{2}) dx$.

7.Use integration by parts to find the integrals (a) $\int x^2 \ln x dx$ and (b) $\int x^2 e^x dx$.

8.Find the integral $\int \frac{x^4+6x^3+10x^2+x}{x^2+6x+10} dx$.

9.Decide the convergence or divergence of the following improper integrals:

(a) $\int_1^{\infty} \frac{(\ln x)^2}{x} dx$. (b) $\int_0^{\infty} \frac{1}{1+4x^2} dx$

10.Decide whether the following infinite series converge or diverge. Justify your answer in each case and state the test that you are using.

(a) $\sum_{k=1}^{\infty} \frac{1}{k^k}$. (b) $\sum_{k=1}^{\infty} \frac{c^k}{k!}$.

11.Find the sum of the series $\sum_{k=1}^{\infty} (\frac{1}{3^k} - \frac{2}{4^{k-1}})$.

12.Decide whether the series $\sum_{k=1}^{\infty} \frac{(-1)^k \cdot 12k}{k^2+1}$ converges absolutely , conditionally or is divergent.

13.Find the interval of convergence of the power series $\sum_{k=1}^{\infty} \frac{5^k (x-1)^k}{k^2}$.

14. Find Taylor series of $f(x) = e^x$ at $x_0 = 1$ and find the interval of its convergence.

15. Let $f(x) = \frac{1}{1-2x}$. Find the Maclaurin series of functions $f(x)$ and $f'(x)$. State the results using sigma notation.

16. Find the area of the region outside the cardioid $r = 2 - 2\cos\theta$ and inside circle $r = 4$.