

Final Exam
Math 157 – Calculus II
Spring, 2002

Please do all problems. Points are written to the left of each problem.

- 10 pts 1. The base of a solid is the region bounded by the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$. Find the volume of the solid if the cross sections perpendicular to the x -axis are equilateral triangles. (Hint: The area of an equilateral triangle of side s is $\frac{\sqrt{3}}{4}s^2$.)
- 10 pts 2. Find the volume generated when the region bounded by the graphs of $y = 2x - x^2$ and $y = 0$ is revolved around the y -axis.
- 10 pts 3. A force of 2 Newtons was required to stretch a spring from its natural length of 30 centimeters to a length of 40 centimeters. How much work was done in stretching the spring to that length?
- 30 pts 4. Evaluate the following integrals:
- (a) $\int \frac{x-3}{x^2(x+1)} dx$
- (b) $\int \frac{x^2}{1+x^2} dx$
- (c) $\int \sin^3 x \sec x dx$
- 10 pts 5. Determine whether the improper integral $\int_1^2 \frac{1}{\sqrt[3]{x-1}} dx$ converges or diverges, giving a reason for your answer. If it converges, find its value.
- 30 pts 6. Determine whether the following series converge or diverge, giving reasons for your answers.
- (a) $\sum_{k=1}^{\infty} \frac{(k+1)(k+2)}{k(k-2)(k+3)}$
- (b) $\sum_{k=1}^{\infty} \left(\frac{2k+1}{3k-4}\right)^k$
- (c) $\sum_{k=2}^{\infty} \frac{1}{k \ln k}$

- 20 pts 7. Determine whether the following series converge absolutely, converge conditionally, or diverge, giving reasons for your answers.

(a)
$$\sum_{k=0}^{\infty} (-1)^k \frac{k}{k^2 + 2}$$

(b)
$$\sum_{k=0}^{\infty} \frac{(-1)^{k+1}}{(k+1)(k+2)}$$

- 30 pts 8. Determine the interval of convergence of each of the following power series.

(a)
$$\sum_{k=0}^{\infty} \frac{k}{2k+1} (x-2)^{2k+1}$$

(b)
$$\sum_{k=1}^{\infty} \frac{(-1)^k}{\sqrt{k}} (x+3)^k$$

(c)
$$\sum_{k=0}^{\infty} \frac{2^k}{(2k)!} x^k$$

- 10 pts 9. Find the MacLaurin series expansion of $\frac{x}{(1-2x)^2}$.

- 10 pts 10. Find the Taylor series expansion of $\sin(\pi x)$ around the point $x = 1$.

- 10 pts 11. Evaluate $\int_0^1 \sin(x^3) dx$ to three decimal point accuracy. (You will probably find it helpful to recall that for all x , $\sin x = \sum_{k=0}^{\infty} (-1)^k \frac{x^{2k+1}}{(2k+1)!}$.)

- 10 pts 12. Let C be the curve parametrized by the equations

$$x(t) = t^2 - 2t$$

$$y(t) = t^3 - 3t^2 + 2t.$$

Find the points at which the curve has a vertical tangent.

- 10 pts 13. Find the area inside the graph of $r = 4$ and to the right of the graph of $r = 2 \sec \theta$.